11 th lecture

QUANTUM CHROMODYNAMICS

– part 1 –

QCD Lagrangian

the theory of strong interactions
– quantum chromodynamics or QCD
is similar to spinor electrodynamics (QED),
except that

– gauge group is not $U(1)$ but $SU(3)$ (non-Abelian)
– quarks carry e.m. charge but also color charge
(fundamental rep. of $SU(3)$)
– gluons are e.m. neutral but carry color charge
(adjoint repres. of $SU(3)$)

$\rightsquigarrow$ 6 quark flavors, each in 3 colors, 8 gluon species

- generalize the QED Lagrangian

$$\mathcal{L}^{QED}_{fermi} = i\bar{\Psi}\gamma^\mu D_\mu \Psi - m\bar{\Psi}\Psi \quad \text{with } D_\mu = \partial_\mu + ieA_\mu$$

invariant under $A_\mu \mapsto A_\mu + \partial_\mu \lambda$ & $\Psi \mapsto e^{-ie\lambda}\Psi \rightsquigarrow U(1)$

- replace electron field Ψ by quark field q_j, $j=1,2,3$

SU(3) gauge transformation on $q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$ ← Dirac spinor

$$q(x) \longmapsto \Omega(x)\, q(x) \quad \& \quad \bar{q}(x) \longmapsto \bar{q}(x)\, \Omega^\dagger(x)$$

where $\Omega(x) \in SU(3)$, $q(x)$ in fund. repres. of SU(3)

i.e. unitary 3x3 matrix, $\Omega\Omega^\dagger = \mathbb{1}$, $\det \Omega = 1$

– invariant mass term is simple: $-m\bar{q}q = -m\bar{q}^j q_j$

– kinetic term: free part must be $i\bar{q}^j \gamma^\mu \partial_\mu q_j$

to make this locally SU(3) invariant introduce cov. derivative

call g = strong (color) charge

$$(D_\mu)_j{}^k = \delta_j{}^k \partial_\mu + g\,(t_a)_j{}^k A^a_\mu(x)$$ 3x3 matrix, $t^a \to su(3)$ $(j,k=1,2,3)$ $(a=1,\dots,8)$

$$\hat{D}_\mu = \mathbb{1}\,\partial_\mu + g\,\hat{A}_\mu\,, \quad \hat{A}_\mu = t_a A^a_\mu$$ antihermitian traceless 3x3 matrix

$\to$ $\hat{A}_\mu$ lives in $su(3)$ Lie algebra

- need the gauge trsfm. of non-Abelian $\hat{A}_\mu$:

$$\hat{A}_\mu(x) \longmapsto \Omega(x)\left(\hat{A}_\mu(x) + \frac{1}{g}\partial_\mu\right)\Omega^\dagger(x)$$

this makes $\hat{D}_\mu$ transform as a tensor:

$$\hat{D}_\mu \longmapsto \Omega(x)\,\hat{D}_\mu\,\Omega^\dagger(x)$$

$$\hat{D}_\mu q(x) \longmapsto \Omega(x)\,\hat{D}_\mu\,\Omega^\dagger(x)\,\Omega(x)\,q(x) = \Omega(x)\,\hat{D}_\mu q(x) \quad \checkmark$$

check:

$$(\mathbb{1}\partial_\mu + g\hat{A}_\mu)\,q \mapsto (\mathbb{1}\partial_\mu + g\,\Omega\hat{A}_\mu\Omega^\dagger + \Omega\,\partial_\mu\Omega^\dagger)\,\Omega q$$

$$= \Omega\Big(\partial_\mu + g\hat{A}_\mu + \underbrace{\Omega^\dagger(\partial_\mu\Omega) + (\partial_\mu\Omega^\dagger)\Omega}_{=\,\partial_\mu(\Omega^\dagger\Omega)\,=\,0\ \checkmark}\Big)\,q$$

$$= \Omega\,(\partial_\mu + g\hat{A}_\mu)\,q$$

- Abelian limit: $SU(3) \to U(1)$

$\Omega \to e^{-ie\lambda}$, $g \to -e$, $q \to \psi$, $\hat{A}_\mu \to -iA_\mu$

$\rightsquigarrow \hat{D}_\mu \to \partial_\mu + ieA_\mu$, $\Omega q \to e^{-ie\lambda}\psi$, $i\Omega(\hat{A}_\mu + \frac{1}{g}\partial_\mu)\Omega^\dagger \to A_\mu + \partial_\mu\lambda$

- generalize Maxwell term

$\mathcal{L}^{QED}_{Maxwell} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ with

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{1}{ie}[D_\mu, D_\nu]$

have generalized $D_\mu \to \hat{D}_\mu$, so copy this relation ($ieA_\mu \to g\hat{A}_\mu$)

$\hat{F}_{\mu\nu} := \frac{1}{g}[\hat{D}_\mu, \hat{D}_\nu] = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g[\hat{A}_\mu, \hat{A}_\nu]$

field strength also takes value in $su(3)$ Lie algebra $\rightsquigarrow$
expand in $SU(3)$ generators t_a: $\hat{F}_{\mu\nu} = F^a_{\mu\nu} t_a$ $\rightsquigarrow$

$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ | $f^{abc} = SU(3)$ structure constants |

under gauge transformations:

$\hat{F}_{\mu\nu} \longmapsto \Omega \hat{F}_{\mu\nu} \Omega^\dagger$ not invariant, but in adjoint representation but color traces are gauge invariant, such as

$$\mathcal{L}^{QCD}_{gluon} = \frac{1}{2} tr\left(\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}\right) = -\frac{1}{4} F^a_{\mu\nu}F^{\mu\nu a} \qquad \text{YM 1954}$$

- full QCD Lagrangian density (each q_f is a triple $\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}_f$ of Dirac spinors)

$$\mathcal{L}^{QCD} = \frac{1}{2} tr\left(\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}\right) + \sum_{f=1}^{6} \left\{ i\bar{q}_f \gamma^\mu \left(\partial_\mu + g\hat{A}_\mu\right) q_f - m_f \bar{q}_f q_f \right\}$$

QCD Feynman rules

- similar to those of spinor QED, but including the SU(3) generators $(t_a)_j{}^k$ & structure constants $f_{abc} = (t_b^{adj})_a{}^c$

$$e\bar{\psi}\gamma^\mu A_\mu \psi \longrightarrow g\,\bar{q}^j \gamma^\mu A^a_\mu (t_a)_j{}^k q_k$$

"quark-gluon vertex" 

new feature: 3- & 4-gluon vertex  & 

$$\frac{1}{2} tr\,\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} \ni tr\left\{ (\partial_\mu \hat{A}_\nu)(\partial^\mu \hat{A}^\nu - \partial^\nu \hat{A}^\mu) + 2g(\partial_\mu \hat{A}_\nu)[\hat{A}^\mu, \hat{A}^\nu] + g^2 [\hat{A}_\mu, \hat{A}_\nu][\hat{A}^\mu, \hat{A}^\nu] \right\}$$

- these rules suffice for tree diagrams but for loops the problem is virtual gluons

like for photons, $\exists$ two unphysical polarizations

in basis $p = (k, 0, 0, k)$: [physical: $p \cdot \varepsilon = 0$]

"temporal" $\varepsilon^{(T)}_\mu = (1, 0, 0, 0)$ & "longitudinal" $\varepsilon^{(L)}_\mu = (0, 0, 0, 1)$

in QED: $\varepsilon^{(T)}_\mu$ & $\varepsilon^{(L)}_\mu$ contributions cancel in each loop ($\partial_\mu j^\mu = 0$)

in QCD: unphysical virtual gluon polarizations don't cancel ($D_\mu j^\mu = 0$)

2 ways out
- Lorentz-noninv. gauge $\rightarrow$ transverse gluon prop $\rightarrow$ complicated
- Lorentz-inv. gauge $\rightarrow$ add "ghost fields" to compensate (Faddeev & Popov 1967)

Asymptotic freedom & confinement

• remember lecture 5:

effective strong coupling $g^2(\overset{\text{energy scale}}{\mu})$ <u>de</u>creases with energy μ

$\rightarrow$ at high energies, small coupling $\rightarrow$ perturbation theory works

"asymptotic freedom": quarks quasi-free, jet production

• low-energy behavior: g^2 large ($\to \infty$ at $\mu \to \Lambda_{QCD}$)

[but one-loop approx. fails there, pert. thy breaks at $\mu \lesssim \frac{1}{2}$ GeV]

• heavy quark-antiquark pair: consider $t\bar{t}$ boundstate

at small distance $r \ll \Lambda_{QCD}^{-1}$ attraction by virtual one-gluon exchange

↳ Coulomb-like potential $V_{q\bar{q}}(r) = -c_F \dfrac{g^2(\mu \sim 1/r)}{4\pi r}$

with $c_F \mathbb{1} = -t_a t_a = \frac{4}{3}\mathbb{1}$

at larger distances $r \gtrsim \Lambda_{QCD}^{-1}$ other exchanges relevant

↳ Coulomb formula no longer valid

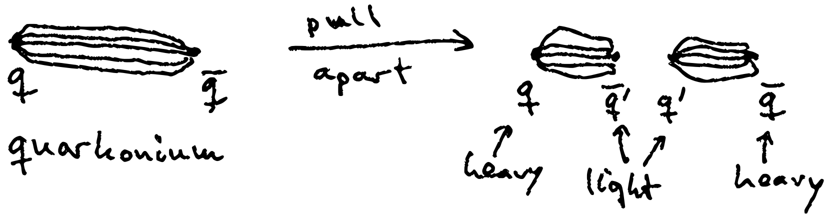
conjecture: linear growth of V | combine in "funnel potential"

$V_{q\bar{q}}(r) \sim \sigma r, \quad \sigma > 0$

- unfortunately, we cannot even prove that $V_{q\bar{q}}(r\to\infty)=\infty$ (confinement: no colored asymptotic states)

- heuristic justification for linear growth:

  gluon self-interaction $\rightsquigarrow$ chromoelectric field is concentrated in a tube $\rightarrow$ "QCD string"



string breaking to meson pairs ($q\bar{q}$)

  estimate & fits $\rightsquigarrow$ constant linear string tension $\sigma \approx 0.9\,\frac{\text{GeV}}{\text{fm}}$

- heavy quarkonium = $q\bar{q}$ + gluon string is color-neutral

  for heavy quarks most energy is in quark masses $\rightsquigarrow m_{t\bar{t}} \approx 2m_t$

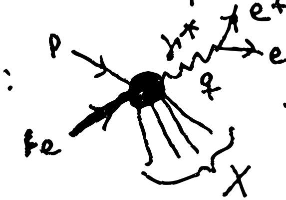
  for lighter quarks significant energy is in gluon tube $\rightsquigarrow m_{u\bar{u}} \gg 2m_u$

November revolution & quarkonium

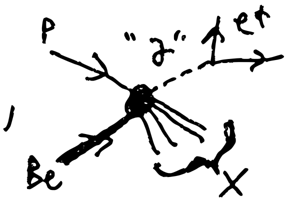
- historical highlight: sudden paradigm change by striking experimental evidence

11 Nov 1974: 2 independent groups saw clear signal

– Sam Ting @ BNL:



expected,



seen

– Burton Richter @ SLAC: $e^+e^- \to e^+e^-$, seen: $e^+e^- \to$ "ψ" $\to e^+e^-$

same sharp peak at invariant mass² $M^2_{e^+e^-} = (p_+ + p_-)^2 \approx 3.1\,\text{GeV}$

⇝ at this energy a resonance occurs: $\dfrac{1}{(p_+ + p_-)^2} \longrightarrow \dfrac{1}{q^2 - m^2_{J/\psi}}$

finite width Γ of peak due to experimental imprecision

intrinsic width $\Gamma \sim \hbar/\tau$ $\quad (\Delta E \cdot \Delta t \sim \hbar)$ gave $\Gamma_{J/\psi} \approx 90\,\text{keV}$

⇝ simultaneous indep. discovery ⟶ Nobel prize 1976

$\Updownarrow$

⇝ only particle with a double name "long" $\tau \approx 7 \cdot 10^{-21}\,\text{s}$

- what was so important about the J/ψ?

J/ψ is a special long-lived hadron, not made of u, d or s

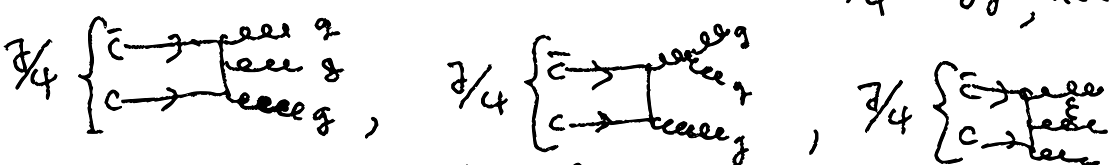
$\rightsquigarrow$ a bound state of a new quark (c) with its antiquark ($\bar{c}$)

first experimental verification of heavy quarkonium

- properties of J/ψ

spin 1 $\sim$ "orthocharmonium" $|c\bar{c}\rangle \times |\uparrow\uparrow\rangle \times |\text{color}\rangle$

decays: C conjugation inv. forbids $J/\psi \to gg$, hence $J/\psi \to ggg$



$$\tau^{-1} = \Gamma = \int dw = \frac{160(\pi^2-9)}{81\, m^2}\,\alpha_s^3\,|\psi(0)|^2 + \mathcal{O}(\alpha_s^4)$$

$[\alpha_s = g^2/4\pi]$ model after $e^+e^- \to q\bar{q}$ small since $\alpha_s(3\,\text{GeV}) \simeq 0.25$

$\rightsquigarrow$ Γ fits with experiment

- paracharmonium: $|c\bar{c}\rangle \times |\uparrow\downarrow\rangle \times |\text{color}\rangle = \eta_c \to gg$

$m \simeq 2.98$ GeV ($m_{J/\psi} - m_{\eta_c}$ = hyperfine splitting), in $J/\psi \to \eta_c \gamma$

$\Gamma \sim \alpha_s^2 \simeq 30$ MeV $\gg \Gamma_{J/\psi}$

- charmonium spectroscopy

in analogy with e^+e^- or hydrogen, many excited states $c\bar{c}$ most observed in experiment $\rightsquigarrow$ spectroscopic quantum #s

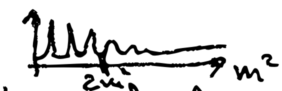
above $m \simeq 3.75$ GeV the widths are suddenly larger:

above threshold for decay $(c\bar{c}) \to (c\bar{q})(q\bar{c})$

u or d

into D-mesons (charmed mesons)

↳ have $m_D \simeq 1.87$ GeV



soon above ~4 GeV spectrum becomes continuous

- repetition for bottomonium (& probably toponium)

$b\bar{b} = \Upsilon$ with $m_\Upsilon \simeq 9.46$ GeV at Fermilab 1977 Leon Lederman

- why the excitement in 1974?
  - quarks were hypothetical, their existence was not accepted
  - confinement explanation for non-observation seemed strange
  - but theoretical discovery of asymptotic freedom became known
  - charmonium spectrum had a natural interpretation via $c\bar{c}$ bound states

$\Rightarrow$ today, no more doubts that quarks exist and are described by QCD